A Naphthalene Sublimation Study on Heat/Mass Transfer for Flow over a Flat Plate

Jong-Hark Park
Korea Atomic Energy Research Institute, Dukjin-dong, 150 Yusung-gu, Taejeon 305-353, Korea
Seong-Yeon Yoo*
Department of Mechanical Design Engineering, Chungnam National University, Guong-dong, Yusung-gu, Taejeon 305-764, Korea

It is important to completely understand heat/mass transfer from a flat plate because it is a basic element of heat/mass transfer. In the present study, local heat/mass transfer coefficient is obtained for two flow conditions to investigate the effect of boundary layer using the naphthalene sublimation technique. Obtained local heat/mass transfer coefficient is converted to dimensionless parameters such as Sherwood number, Stanton number and Colburn j-factor. These also are compared with correlations of laminar and turbulent heat/mass transfer from a flat plate. According to experimental results, local Sherwood number and local Stanton number are in much better agreement with the correlation of turbulent region rather than laminar region, which means analogy between heat/mass transfer and momentum transfer is more suitable for turbulent boundary layer. But average Sherwood number and average Colburn j-factor representing analogy between heat/mass transfer and momentum transfer are consistent with the correlation of laminar boundary layer as well as turbulent boundary layer.

Key Words: Naphthalene Sublimation, Heat/Mass Transfer Analogy, Flat Plate

Nomenclature

C_f : Skin friction coefficient, (dimensionless)
$\overline{C}_{f,L}$: Average skin friction coefficient at $x=L$, (dimensionless)
$C_{f,x}$: Local skin friction coefficient, (dimensionless)
D_{iff} : Mass diffusion coefficient of naphthalene in the air, (m^2/sec)
h : Heat transfer coefficient, (W/m^2k)
h_m : Mass transfer coefficient, (m/sec)
j_H : Colburn j factor for heat transfer
j_m : Colburn j factor for mass transfer
L : Distance from leading edge to last measured point location in flow direction, (m)
Nu : Nusselt number, (dimensionless)
$P_{v,w}$: Vapor pressure of naphthalene, (Pa)
Pr : Prandtl number, (dimensionless)
Re_x : Reynolds number based on distance from leading edge, (dimensionless)
Re_L : Reynolds number at $x=L$
$Re_{L=200mm}$: Reynolds number based on distance from leading edge to $x=200$ mm
Sc : Schmidt number, (dimensionless)
Sh : Sherwood number, (dimensionless)
$\overline{Sh}_L$: average Sherwood number at $x=L$ (dimensionless)
Sh_x : Sherwood number based on distance from the leading edge (dimensionless)
St : Stanton number for heat transfer, (dimensionless)
St_m : Stanton number for mass transfer, (dimensionless)
St_x : Stanton number for heat transfer based on the distance from the leading edge,

* Corresponding Author,
E-mail : syyooh@cnu.ac.kr
TEL : +82-42-821-6646; **FAX** : +82-42-822-7366
Department of Mechanical Design Engineering, Chungnam National University, Guong-dong, Yusung-gu, Taejeon 305-764, Korea. (Manuscript **Received** February 19, 2004; **Revised** April 8, 2004)

(dimensionless)

$St_{m,x}$: Stanton number for mass transfer based on the distance from the leading edge (dimensionless)

T : Temperature, (K)

Δt : Sublimation depth, (m)

u_∞ : Free stream air velocity, (m/s)

Greek symbols

δ : Momentum boundary layer thickness

δ_M : Mass concentration boundary layer thickness

δ_T : Thermal boundary layer thickness

ρ_s : Density of solid naphthalene, (kg/m^3)

$\rho_{v,w}$: Density of naphthalene vapor, (kg/m^3)

$\Delta\tau$: Run time in wind tunnel, (sec)

1. Introduction

If the temperature profile is identical to the velocity profile for a flow over a flat plate (Prandtl number, Pr, is unity), the Reynolds analogy is expressed as

$$\frac{Nu}{RePr}=\frac{h}{\rho c_p u_\infty}=St=\frac{C_f}{2} \tag{1}$$

where St is the dimensionless Stanton number, and h is the heat transfer coefficient. The skin friction coefficient C_f is written in terms of wall shear force τ_w as follows.

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} \tag{2}$$

Since the Reynolds analogy is validated only for Pr=1, Colburn suggested a minor change of the Reynolds analogy form which can be applied to cases when $0.5<Pr<50$. The equation named either the Colburn analogy or the Reynolds-Colburn analogy designating relation between the fluid friction and the heat transfer for laminar flow over a flat plate is given by

$$\frac{C_{f,L}}{2}=St_L Pr^{2/3}\equiv j_H \quad \text{for } 0.6<Pr<60 \tag{3}$$

where j_H designates the Colburn j-factor for heat transfer.

The skin friction factor for laminar flow is obtained from the following equation.

$$C_{f,x}=\frac{0.664}{\sqrt{Re_x}} \tag{4}$$

Many correlations of the skin friction coefficients for turbulent boundary layer are summarized by Schlichting (1979). Equation (5) is in a quite good agreement with experiments for Reynolds number up to several million, but then becomes increasingly lower than the experimental data for higher Reynolds number.

$$\frac{C_{f,x}}{2}=\frac{0.0296}{Re_x^{1/5}} \quad \text{for } 5\times10^5<Re<10^7 \tag{5}$$

According to above discussion, the Eq. (3) can be rewritten as :

$$St_x Pr^{2/3}=\frac{0.332}{Re_x^{1/2}} \quad \text{for laminar flow, and} \tag{6}$$

$$St_x Pr^{2/3}=\frac{0.0296}{Re_x^{1/5}} \quad \text{for } 5\times10^5<Re<10^7 \tag{7}$$

If the Schmidt number, Sc, corresponding to the Prandtl number in heat transfer, is unity (concentration profile is identical to velocity profile), Reynolds assumption can be extended to the transfer mechanism for momentum and mass. Therefore, it can be assumed that the mechanism of heat transfer is very analogous to that of mass transfer.

According to this assumption, when Pr is equal to Sc, alternative correlations are obtained as followings by replacing Pr of Eq. (3), (6) and (7) with Sc.

$$\frac{Sh}{ReSc}=\frac{h_m}{u_\infty}=St_m=\frac{C_f}{2} \tag{8}$$

$$\frac{C_{f,L}}{2}=St_{m,L}Sc^{2/3}\equiv J_m \quad \text{for } 0.6<Sc<3000 \tag{9}$$

$$St_{m,x}Sc^{2/3}=\frac{0.332}{Re_x^{1/2}} \quad \text{for laminar flow, and} \tag{10}$$

$$St_{m,x}Sc^{2/3}=\frac{0.0296}{Re_x^{1/5}} \quad \text{for } 5\times10^5<Re<10^7 \tag{11}$$

where St_m (the mass transfer Stanton number), Sh, and h_m correspond to those of heat transfer. Equation (8) is called as the Chilton-Colburn analogy, and j_m designates the Colburn j-factor for mass transfer.

Above correlations of the heat/mass transfer from a flat plate were derived from skin friction coefficient obtained analytically or experimentally (Schlichting, 1979) based on the Reynolds-Colburn analogy. Consequently, there have been few researches (Yoo et al., 1993 ; Goldstein et al., 1990) to measure directly local heat/mass transfer coefficient from a flat plate which can be compared with the correlations of heat/mass transfer.

In the present study, the frequently used heat/mass transfer correlations for flat plate are compared with experimentally measured local heat/mass transfer coefficient. Also the influence of momentum boundary layer development on heat/mass transfer is investigated.

2. Experimental Apparatus and Data Reduction

2.1 Experimental apparatus

The experimental apparatus consists of a wind tunnel, a naphthalene casting facility and an automated sublimation depth measurement system. An open-circuit, blowing-type wind tunnel is used, which has square test section of 300 mm wide ×330 mm high. The air-speed of the wind tunnel is controlled by an inverter and the free-stream turbulence intensity is less than 0.5% over the entire range of speed.

The automated sublimation depth measurement system consists of a depth gage along with a signal conditioner, two stepping motor-driven X-Y traversing table, a hardware unit for motor control, and data acquisition system. The depth gage used to measure the naphthalene surface profile is a linear variable differential transformer (LVDT), which has ±0.254mm (±0.01 in) linear range and 25. 4 nm (1 μ in) resolution. It is connected to a signal conditioner which supplies current to the LVDT for excitation, converts the AC signal output of the depth gage to a DC voltage. For precise and automatic positioning of the LVDT on the test-piece, the LVDT and test-pieces are tightly mounted on the X-Y traverse table, and two stepper motors move the table with 0.0254 mm (0.001 in) per step. The data acquisition system equipped with personal computer controls the movements of stepping motors and gets the measured data from the signal

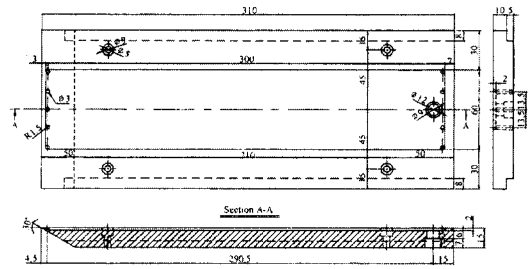


Fig. 1 Schematic of the flat plate test piece (dimension : mm)

conditioner.

The test-piece is made of aluminum and has a groove (300 mm length×60 mm width×2 mm depth) to cast naphthalene as shown in Fig. 1. Flat plate has a sharp edge machined at the angle of attack 30° to prevent boundary layer or its separation from developing suddenly.

2.2 Data reduction

Using the automated sublimation depth measurement system, surface profiles are measured on the naphthalene surface before and after the exposure in the wind tunnel. Then, the mass transfer coefficients are obtained from the following equation

$$h_m=\rho_s\Delta t/\rho_{v,w}\Delta\tau \tag{12}$$

where ρ_s is the density of the solid naphthalene, $\rho_{v,w}$ is the naphthalene vapor density on the surface, Δt is the net sublimation depth, and $\Delta\tau$ is the total exposure time in the wind tunnel. Total naphthalene sublimation depth is calculated from the difference in naphthalene surface elevations before and after the exposure in the wind tunnel, and the excess sublimation due to natural convection during the sublimation depth measurement period is subtracted from the total sublimation. Naphthalene is sublimated approximately 0.05 mm for an hour exposure, consequently, geometry change due to sublimation doesn't affect the flow. The empirical equation of Ambrose et al. (1975) is used to calculate the naphthalene vapor pressure from the measured naphthalene surface temperature, and naphthalene vapor density on the surface is then evaluated using the ideal gas law. The measured mass transfer coefficients are presented in the dimensionless form, that is the Sherwood number defined as follows

$$Sh = h_m x / D_{iff} \tag{13}$$

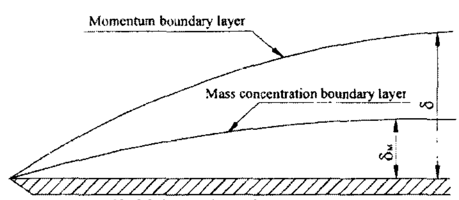
Characteristic length x means distance from the leading edge of flat plate, and the mass diffusion coefficient of naphthalene in air D_{iff} is calculated by Cho's correlation (Cho, 1989). According to the uncertainty analysis recommended by Kline and McClintock (1953), the estimated error of Sherwood number is about 6.11% as shown in the following equation (Park, 2003 ; Yoo et al., 2003).

$$\begin{aligned} \frac{\Delta Sh}{Sh} &= \sqrt{\left(\frac{\Delta \rho_s}{\rho_s}\right)^2 + \left(\frac{\Delta \Delta t}{\Delta t}\right)^2 + \left(\frac{\Delta \rho_{v,w}}{\rho_{v,w}}\right)^2 + \left(\frac{\Delta \Delta \tau}{\Delta \tau}\right)^2 + \left(\frac{\Delta D_{iff}}{D_{iff}}\right)^2} \\ &= \sqrt{0.011^2 + 0.02985^2 + 0.04266^2 + 0.00278^2 + 0.03^2} \\ &= 0.0611 \end{aligned} \tag{14}$$

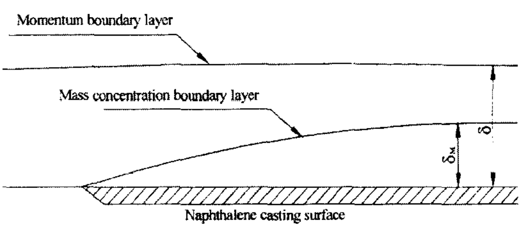
3. Results and Discussion

Mass transfer from a flat plate is investigated for two flow conditions : flat plate in parallel flow with and without un-sublimated (un-heated) starting length. Flat plate without un-sublimated (un-heated) starting length means that a mass concentration boundary layer (thermal boundary layer in heat transfer) will originate together with the momentum boundary layer at the leading edge of the plate as shown in Fig. 2(a). That is to say, the momentum boundary layer and the concentration boundary layer develop simultaneously. On the contrary, the other one with un-sublimated (un-heated) starting length means that momentum boundary layer is considerably developed before the mass concentration boundary layer as can be seen in Fig 2(b), so the concentration boundary layer will be developed in the momentum boundary layer.

Forty-one data in the streamwise direction and eleven data in the spanwise direction are measured to obtain local mass transfer coefficients. Each points plotted in graphs are averaged values for spanwise nine points (two points in both sides are omitted for reference). The first measurement point is located at 2 mm downstream from the leading edge.



(a) Flat plate without un-sublimated starting length



(b) Flat plate with un-sublimated length condition

Fig. 2 Schematics of the two flow conditions

3.1 Flat plate without un-sublimated starting length

This boundary condition, a constant density surface, corresponds to a constant temperature surface in heat transfer. This flow condition can be met by installing the test piece on the mounting board as shown in Fig. 3.

Figure 4 shows the variation of local mass transfer coefficient along the distance from the leading edge when the free stream velocity varies from 2 m/s to 20 m/s. As the distance from the leading edge increases, the local mass transfer coefficient rises up rapidly, and then decreases steeply for a while. After that, h_m decreases gradually with distance.

When parallel flow goes over a constant temperature (or a constant density) flat plate, the

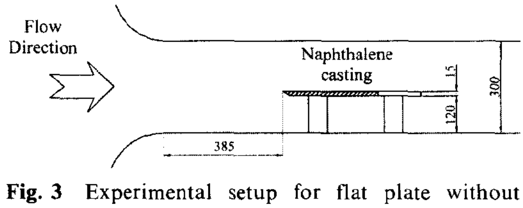


Fig. 3 Experimental setup for flat plate without un-sublimated starting length (dimension : mm)

boundary layer is initially laminar where the heat/mass transfer coefficient decreases rapidly. But at a certain distance from the leading edge, transition to turbulence occurs. At this time, the heat/mass transfer coefficient shows a sudden increment. Further going ahead, the flow becomes fully turbulent where the heat/mass transfer coefficient decreases along the flow direction as the boundary layer becomes thicker. Figure 5 illustrates schematically the variation of momentum boundary layer and local heat transfer coefficient. In comparison of the variation trend of the mass transfer coefficient of Fig. 4 with h_m of Fig. 5, these are very similar with each other with the exception of the laminar region. Observing the local Sherwood number in Fig 4, a small rise can be seen near the leading edge, which seems to indicate the end of the laminar region. Generally, to make and keep up a laminar flow in experiment is very difficult, because it may disappear very quickly. So, it is hard to obtain the mass transfer coefficient in the laminar region of leading edge.

The location of maximum h_m moves towards the leading edge with the free-stream velocity until $u_\infty = 8$ m/s, then it backs to downstream with further increase of u_∞. It seems that the transition happens gently at low speed, and grows

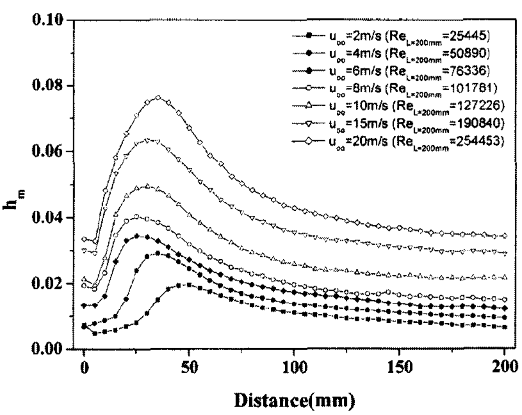


Fig. 4 Local mass transfer coefficient variation along the distance from the leading edge of flat plate without un-sublimated starting length

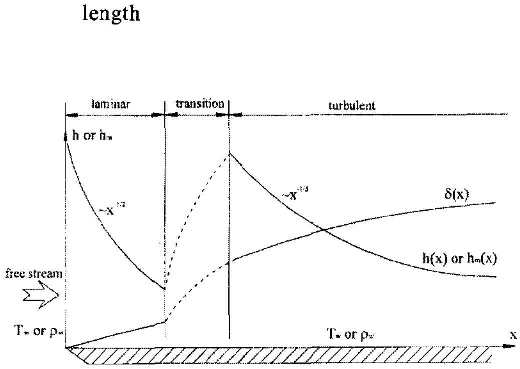


Fig. 5 Variation of momentum boundary layer thickness and the local heat/mass transfer coefficient for flow over a constant temperature/density flat plate

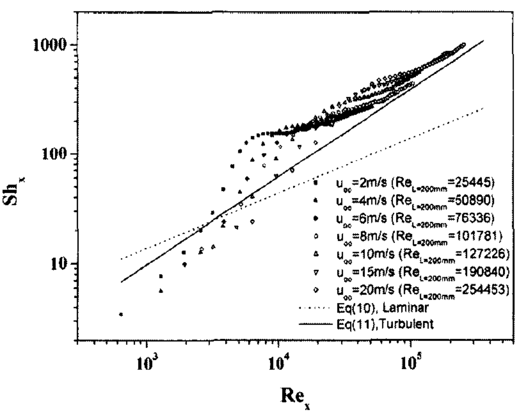


Fig. 6 Local Sherwood number variation of flat plate without un-sublimated starting length vs. Re for various free stream velocity

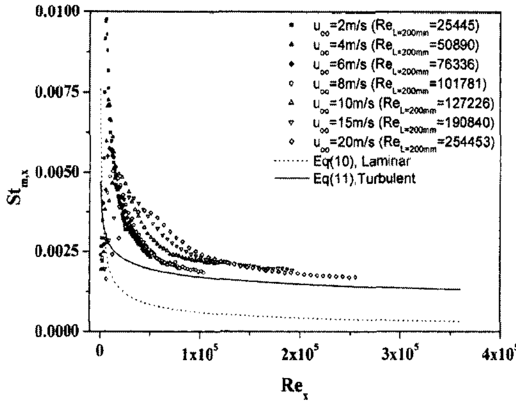


Fig. 7 Local mass transfer Stanton number variation of flat plate without un-sublimated starting length vs. Re for various free stream velocity

at high speed. But it is not clear why the maximum point backs to downstream.

The experimental results for the variation of local Sherwood number with Re_x are shown flow in Fig. 6 and compared with the correlations of heat/mass transfer for laminar and turbulent flows. As can be seen in this figure, experimental results do not agree with the correlations in the transition region. However, in the turbulent region, all of them approach to turbulent heat/mass transfer correlations.

Relation of the mass transfer Stanton number to Re_x is presented in Fig. 7. As mentioned previously, St_m comes close to turbulent heat/mass transfer correlation for high Re_x.

3.2 Flat plate with un-sublimated (un-heated) starting length

To establish an un-sublimated starting length condition, flat plate is installed as shown in Fig 8. Height of the test section is 300 mm which is enough larger than momentum boundary layer thickness (δ) so this experimental condition can be considered as an external flow rather than a duct flow. At the starting point of the naphthalene casting surface, Reynolds number based on a distance from the beginning of test section is about 62,500 at $u_\infty=2$ m/s and 250,000 at $u_\infty=8$ m/s. In general, it is well known that laminar boundary layer on a flat plate goes through transition to turbulent boundary layer when Re_x based on the distance from the leading edge reaches the range of 300,000~500,000. Even a relatively high velocity beyond 8 m/s ($Re_x=$ 218,600) does not satisfy the requirement for fully developed turbulent boundary layer.

The variation of the mass transfer coefficient is shown in Fig. 9. Regardless of free stream velocity, the Sherwood numbers decrease very quickly from the leading edge to the downstream, and then maintain flat values. As the free stream velocity increases, the Sherwood number also increases.

From the Fig. 9, two remarkable things are found. At low velocity under 6 m/s, the increment of Sherwood number with increasing u_∞ is relatively smaller than that of high velocity over 6 m/s. Another point is that the local Sherwood number at $u_\infty=6$ m/s ($Re_x=188{,}000$)

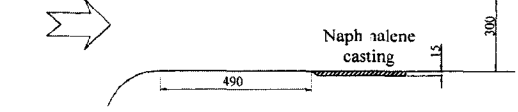


Fig. 8 Experimental setup for flat plate with un-sublimated (un-heated) starting length (dimension: mm)

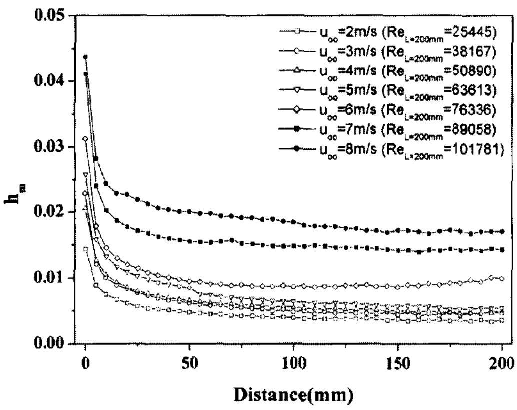


Fig. 9 Local mass transfer coefficient variation along the distance from the leading edge of flat plate with un-sublimated starting length condition

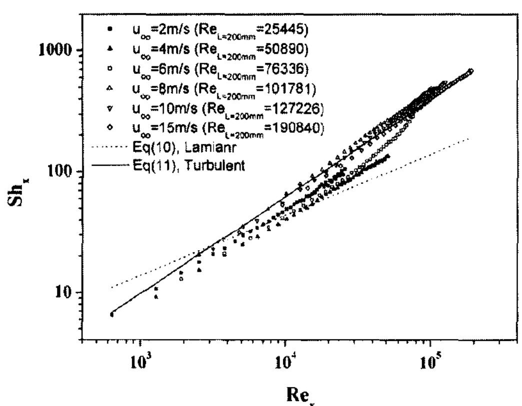


Fig. 10 Local Sherwood number variation of flat plate with un-sublimated starting length vs. Re for various free stream velocity

rises gradually with distance from the leading edge, which is also considered as result of the boundary layer transition from laminar to turbulent.

Figure 10 shows the relation between the Sherwood number and the Reynolds number in a logarithmic scale where both Sh_x and Re_x are based on the distance from the leading edge of test plate. Sh_x increases linearly as Re_x increases. The experimental results are compared with the correlations. At high flow velocity above 8 m/s, Sh_x is in good agreement with the correlation for turbulent mass transfer. On the contrary, Sh_x at low flow velocity under 6 m/s does not agree with any correlations, except with u_∞=6 m/s.

The relation of the local mass transfer Stanton number $St_{m,x}$ to Re_x are shown in Fig. 11. From the definition, the Stanton number implies that heat/mass transfer mechanism is very closely connected with the momentum transfer whereas the heat/mass transfer coefficient means how well or how much heat/mass is transferred. In Fig. 11, the mass transfer Stanton number obtained from experiment at low velocity under 6 m/s does not agree with the correlations either for the laminar region or for the turbulent region. But at high Reynolds number (meaning high velocity or fully developed turbulent), St_m agrees with the correlation for the turbulent heat transfer. This means that the analogy between the heat/mass transfer and the momentum transfer is more adequate for fully developed boundary layer.

At low velocity under 6 m/s, St_m is different from the laminar correlation for low Re_x, because the flow over a flat plate can not keep laminar. For u_∞=6 m/s, St_m rises gradually to turbulent correlations with increasing Re_x. This means the mass concentration boundary layer goes through transition to turbulent, as previously mentioned.

3.3 Effect of flow condition

As shown in Fig. 12 and 13, the average Sherwood numbers and the Colburn *j*-factors obtained from experiments are compared with well-known correlations of mass transfer and skin friction. The correlation of average Sherwood

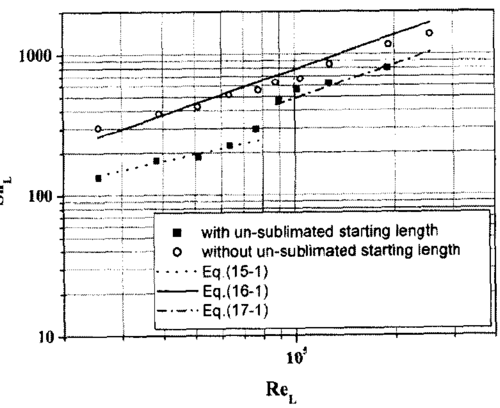


Fig. 12 Variation of average Sherwood number with Re

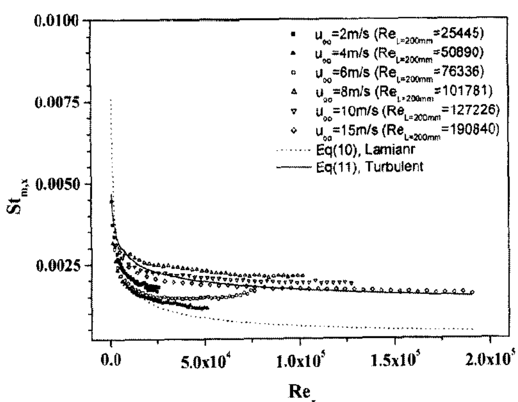


Fig. 11 Local mass transfer Stanton number variation of flat plate with un-sublimated starting length vs. Re for various free stream velocity

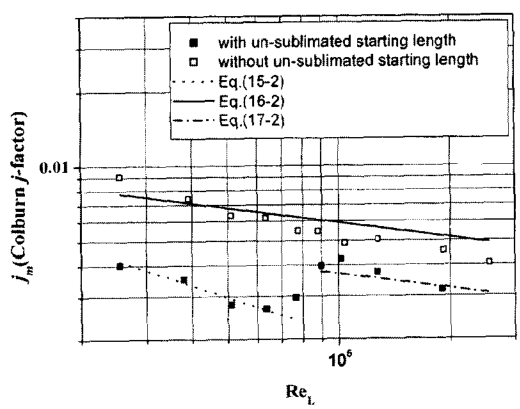


Fig. 13 Variation of Colburn *j*-factor with Re

number can be obtained by integrating local Eqs. (10) and (11). In the same manner, the skin friction coefficient also can be derived from Eqs. (4) and (5).

For the laminar boundary layer condition,

$$\overline{Sh}_L=0.664Re_L^{1/2}Sc^{1/3} \tag{15a}$$

$$\bar{C}_{f,L}/2=0.664Re_L^{-1/2} \tag{15b}$$

For the turbulent boundary layer condition

$$\overline{Sh}_L=0.0592Re_L^{4/5}Sc^{1/3} \tag{16a}$$

$$\bar{C}_{f,L}/2=0.0592Re_L^{-1/5} \tag{16b}$$

There are other correlations of mixed boundary layer condition (Incropera, 1996) which considers laminar and turbulent boundary layers all together, i.e.,

$$\overline{Sh}_L=0.037Re_L^{4/5}Sc^{1/3} \tag{17a}$$

$$\bar{C}_{f,L}/2=0.037Re_L^{-1/5} \tag{17b}$$

The Sherwood number with un-sublimated starting length increases with the Reynolds number, but there is a change of slope with increasing the Reynolds number. At low Reynolds number, experimental results are consistent with correlation of laminar boundary layer condition. However, at Reynolds number over 90,000, they follow he correlation of mixed boundary layer rather than the turbulent boundary layer. From this results, it seems that the transition begins at the range of $Re_x=30,000$ to 50,000. These results are in good agreement with the correlations for laminar and mixed boundary layer.

However, there is not that kind of a change of slope without the un-sublimated starting length. The Sherwood number increases linearly as the Reynolds number rises. It can be thought that the development of concentration boundary layer for developing flow arises very quickly caused by the development of momentum boundary layer, so it seems that transition region locates itself very close to leading edge, as shown in Fig. 4. These are in good agreement with the mass transfer correlation for turbulent flow.

To examine the analogy between momentum and mass transfer, the Colburn j-factor j_m was introduced. The relation of Colburn j-factor to Reynolds number can be seen in Fig. 13. In spite of a small difference between the experiment and the correlation, the variation of Colburn j-factor is very similar to each other. These facts imply that the mass transfer is also very analogous to the momentum transfer.

4. Conclusions

An experimental study to investigate heat/mass transfer from a flat plate with and without un-sublimated starting length has been conducted. The conclusions from this study can be summarized as following.

The local Sherwood number over a flat plate without un-sublimated starting length rise up rapidly due to transition of boundary layer, and then decreases steeply with development of turbulent boundary layer. The location of maximum h_m moves toward the leading edge with the free stream velocity until $u_\infty=8$ m/s, then it backs to downstream with further increase of u_∞. The local Sherwood number for a flat plate with un-sublimated staring length decreases very quickly from the leading edge to downstream and then maintains the flat values. The local Sherwood number and Stanton number are in good agreement with the correlations for turbulent mass transfer beyond $u_\infty=8$ m/s.

From the above conclusions, the analogy between heat/mass and momentum transfer seems to be more adequate for fully developed turbulent flow. In spite of the discrepancy between local value and correlation for laminar, the average Sherwood number and Colburn j-factor are in consistent with correlation for laminar boundary layer as well as turbulent layer.

Acknowledgment

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